

# Reducing Inconsistency in Pairwise Comparisons Using Multi-objective Evolutionary Computing

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**Abstract**—Pairwise comparisons are commonly used to estimate values of preference among a finite set of decision alternatives with regards to intangible factors. Inconsistency within decision making judgments may occur. This work proposes an approach to reducing inconsistency using multi-objective optimization with the objectives of different inconsistency types and judgment modification measures. The approach allows the decision maker to choose both the inconsistency measure(s) and the modification measure(s) employed to suit their needs and attitudes. Utilizing multi-objective optimization allows for a range of possible trade-off solutions to be presented to the decision maker for selection, aiding them in their pursuit of inconsistency reduction. It also enables better understanding of the characteristics of the decision problem and its inconsistency.

**Keywords**—Decision analysis, Inconsistency, Evolutionary computing, Genetic algorithms, Multi-objective optimization

## I. INTRODUCTION

The concept of Pairwise Comparisons (PC) is an important component in many Multi-Criteria Decision Making (MCDM) methods. The Law of Comparative Judgment [1], allows for a separation of concerns by breaking a set of elements into a number of pairs of elements for comparison. Each PC allows the Decision Maker (DM) to consider only a pair of elements and to determine their preference, and strength of preference, between the pair, with respect to an intangible factor.

Given two elements  $x$  and  $y$  we denote that the DM prefers element  $x$  to element  $y$  with the notation  $x \succ y$ . Various numerical scales may be utilized to represent the strength of preference; the most utilized being the Saaty 1-9 Scale [2]. When, for example, element  $x$  is preferred 3 times more than element  $y$ , this can be denoted as  $x \succ y$  with a preference strength of 3. Conversely the reciprocal comparison, that element  $y$  is 3 times less preferred than element  $x$ , may be denoted as  $y \succ x$  with a preference strength of 1/3. If neither element is preferred over the other, the elements are said to be equally preferred, denoted by a 1. An element compared with itself is also said to have equal preference, and denoted with a 1. The examples in Section IV utilize the 1-9 Scale; however any bounded numerical scale could be utilized.

A set of PCs, one for each pairing of elements in a set of elements, along with the self-comparison values and the reciprocal values, can be collated into a two-dimensional

Pairwise Comparison Matrix (PCM). From a PCM a one-dimensional representation of a DM's judgments – a Preference Vector (PV) - can be derived through the use of a Prioritization Method (PM). Many PMs exist for this task, see [4] for a comprehensive discussion of PMs.

The consistency of a PCM is the extent to which its set of judgments are coherent. When there is inconsistency present in a PCM then any PV derived from it will only be an estimate of its information. Consequently, different PMs may derive different PV estimates. The greater the amount of inconsistency present, the more the PV only represents an estimate of the PCM's judgment information. Approximations of highly inconsistent PCMs produce large errors, hence "approximations from such matrixes make little practical sense" [3]. Inconsistency within a PCM of more than a handful of elements is almost inevitable [4] and therefore needs to be considered. Both ordinal and cardinal inconsistencies are important considerations for a DM.

*Ordinal inconsistency* identifies inconsistent information without the strengths of preference of the DM's judgments being considered. For example given a set of 3 elements,  $a$ ,  $b$  and  $c$ : if  $a \succ b$ ,  $b \succ c$  and  $c \succ a$ , then the judgments are intransitive and contradictory ordinal inconsistency is present. The judgments of elements  $a$ ,  $b$ ,  $c$  above represent a 3-way cycle between the elements. A method to define a measure of ordinal inconsistency in a PCM is via the number of 3-way cycles present. Calculating the number of 3-way cycles and its utilization as an objective as part of our approach is discussed in Section III.

*Cardinal inconsistency* identifies inconsistency between a set of judgments taking into account the strength of preference of each judgment. For example, consider a set of 3 elements  $a$ ,  $b$  and  $c$ : if  $a \succ b$  with a preference strength of  $x$  and  $b \succ c$  with a preference strength of  $y$ , then, for the judgment set to be cardinally consistent, the final judgment between elements  $a$  and  $c$  would need to be such that  $a \succ c$  with a preference strength of  $x*y$ . The *Consistency Ratio* (CR) [2] can be utilized to measure the amount of cardinal inconsistency present in a PCM and is discussed in Section III.

This paper proposes an approach to reducing inconsistency using multi-objective optimization (MOO) through the

combination of both inconsistency and judgment modification measure objectives.

The rest of the paper is structured as follows: Section II discusses proposed approaches for tackling inconsistency, along with the rationale for the multi-objective approach; the multi-objective approach is discussed in Section III; examples using the approach are presented in Section IV; finally, conclusions and future work are discussed in Section V.

## II. APPROACHES TO REDUCING INCONSISTENCY

Once inconsistency has been identified, there are various ways it may be tackled. These include (1) getting the DM to review their judgments; (2) automatically altering the judgments in some way; (3) proceeding but attempting to take the inconsistency knowledge into consideration. Our proposed approach is focused upon the second of these.

Most approaches to altering the judgments in a PCM to reduce inconsistency generally focus upon either ordinal or cardinal inconsistency separately and seek to present a single altered PCM to the DM.

A convergence algorithm approach was proposed in [5] that looks to find an altered PCM that has a cardinal inconsistency measure below a threshold, whilst looking to ensure the amount of departure from the original judgments is below given ranges. The values of altered PCM presented are composed of judgment values that fall outside of the original judgment scale utilized. A similar convergence algorithm approach is proposed in [6]. Only cardinal inconsistency is considered there with the aim to find a solution below a cardinal measure threshold with the smallest deviation from the original PCM. The modified PCM judgment values fall outside of the original judgment scale used. An approach that focuses on reducing ordinal inconsistency is proposed in [7]. This approach seeks to reduce the number of 3-way cycles within a PCM via an iterative process of judgment reversals. It seeks to reverse judgments that will result in the maximum reduction of 3-way cycles to arrive at a solution without any 3-way cycles.

Inconsistency reduction utilizing Evolutionary Computing (EC) has been utilized in [8]. However, only cardinal inconsistency is considered and the approach does not utilize MOO. The reciprocal property of PCM judgments is not always maintained in discovered solutions. Similarly [9] utilized EC for the reduction of inconsistency of a PCM; here the PCM and the altered PCM are represented as fuzzy numbers. This approach considers cardinal inconsistency and does not utilize MOO. Likewise [10] utilizes EC to find an altered solution below a cardinal inconsistency threshold. Inconsistency and the change to the PCM are considered via their combination into a single objective function.

A MOO approach has been utilized for the task of deriving PVs from a PCM [11] called TOP (Two Objective Optimization). Here the two objectives are defined as Total Deviation (TD) - a measure of the total distance that the PV weights are from the initial judgments - and the Number of Violations (NV) - a measure of the number of ranking violations between elements of the PV and the corresponding initial judgments. This approach utilizes EC methodology to derive a set of Pareto-optimal trade-off solutions.

Our optimization approach, which also utilizes EC, attempts to give the DM control over the selected objectives and then presents a range of reduced inconsistency solutions to the DM that preserve the original judgment scale.

## III. OVERVIEW OF APPROACH

Our approach is now described beginning with a discussion of the tradeoff nature of its objectives before discussion of the measures of inconsistency and measures of difference. This is followed by discussion of their usage within a MOO framework.

### A. Multi-objective Optimisation (MOO)

Any reduction of inconsistency within a PCM will inevitably result in some change to the PCM. The DM is seeking to simultaneously optimize the conflicting objectives of reducing inconsistency whilst minimizing the amount of change in their judgments. Our approach utilizes a MOO approach to model this scenario with an emphasis upon giving the DM as much flexibility as possible regarding the measures of inconsistency and measures of difference employed.

### B. Measures of Inconsistency

#### 1) Consistency Ratio (CR)

The CR proposed by Saaty [2] is a measure of the amount of cardinal inconsistency present within a PCM. Firstly the eigenvalue of the largest eigenvector of the PCM ( $\lambda$ -max) is calculated. When an order  $n$  PCM is perfectly consistent then  $\lambda$ -max =  $n$ . Next, the *Inconsistency Index (CI)* of the PCM is determined. The division of  $n-1$  allows CI to be indifferent to the value of  $n$ .

$$CI = \frac{(\lambda \max - n)}{(n - 1)} \quad (1)$$

The CR is then found by dividing the CI by the *Average Consistency Index (ACI)* for the order of the PCM. The ACI values represent the average inconsistency found over 50,000 trials of randomly generated matrixes for each PCM order [2]. (These utilized the 1-9 Scale; appropriate ACI estimations would be needed to be employed for a different bounded scale).

$$CR = \frac{CI}{ACI} \quad (2)$$

The lower the CR value, the lower the amount of cardinal inconsistency present in the PCM. CR is employed as a minimization objective within our MOO approach. Saaty [2] further proposed an acceptability threshold value of a PCM's CR value. The threshold is designed to be an indicator as to whether a PCM is consistent enough for a satisfactory PV estimate to be derived. Using this threshold when a PCM has a CR value of 0.1 or less, it is considered to be acceptable.

#### 2) 3-Way Cycles

The number of 3-way cycles present within a PCM is an ordinal measure of inconsistency. The presence of 3-way cycles can be determined via an algorithm proposed in [12]. This can also be utilized to determine the total number of 3-way cycles within a PCM, usually denoted as  $L$ . We only need to consider cycles of 3 elements as it has been shown that

eliminating all 3-way cycles ensures elimination of cycles of higher orders [13]. The number of 3-way cycles is employed as a minimization objective in our MOO approach.

### C. Measures of Difference

Given an original encoded PCM judgment set (O) represented as a set of N encoded judgments  $(o_1, o_2 \dots o_N)$ . The amount of change between O and a second altered PCM judgment set (A) can be calculated using a variety of measures. Each of these measures, which are listed below, can be employed as objectives in our MOO approach.

#### 1) Number of Judgment Violations (NJV)

NJV is a measure of the number of the initial set of PC judgments that have changed, without consideration of the amount of change of each judgment. Where  $\sigma$  evaluates to 0 or 1 for each Boolean evaluation.

$$NJV = \sum_{j=1}^N \sigma(o_j \neq a_j) \quad (3)$$

#### 2) Total Judgment Deviation (TJD)

TJD is a measure of the total amount of change between the original judgments and an altered judgment set. It takes into consideration the amount of preference change between each judgment comparison.

$$TJD = \sum_{j=1}^N \text{abs} |o_j - a_j| \quad (4)$$

A modified version of the TJD measure is the Squared Total Judgment Deviation (STJD). Here the deviations between the corresponding judgments in both sets are squared, so consequently altered judgments with a large alteration in strength will have a greater impact upon the measure's total.

$$STJD = \sum_{j=1}^N (o_j - a_j)^2 \quad (5)$$

#### 3) Number of Judgment Reversals (NJR)

NJR is a measure of the number of judgments from the original set that have been inverted in an altered judgment set. For example, given an original judgment between elements x and y where  $x \succ y$ : if in an altered judgment set it is the case that  $x \prec y$  then a judgment reversal has occurred. This measure also considers half reversals. Half reversals are defined as occurring when a judgment of equal preference is altered to be a judgment of not equal preference or a judgment not of equal preference is altered to be a judgment of equal preference. When using the 1-9 scale we can specify equal preference; greater than equal preference and less than equal preference, with 1, greater than 1 and less than 1 respectively.

$$NJR = \sum_{j=1}^N R_j \quad (6)$$

where

$$R_j \left\{ \begin{array}{l} > 1 \text{ and } a_j < 1 \\ < 1 \text{ and } a_j > 1 \\ = 1 \text{ and } a_j \neq 1 \\ \neq 1 \text{ and } a_j = 1 \\ \text{otherwise} \end{array} \right.$$

### D. MOO Approach to Reducing Inconsistency

Through selection of measures of inconsistency and measures of difference by the DM, our MOO approach seeks to find a set of non-dominated solutions approximating the Pareto front (see below) of the PCM problem with respect to the set of objectives chosen by the DM that most suit their preferences.

Our approach is to find feasible solutions that strive to simultaneously minimize the set of objectives - which is composed of the set of 1 or more inconsistency objectives, and the set of objectives of 1 or more measures of difference. A feasible solution being one that consists of values from the original judgment scale's range.

A solution  $S_1$  is said to dominate another solution  $S_2$  if for the set of the objectives *Obj* it has a greater objective value for at least one objective and no worse objective values for any of the other objectives.

$$\text{Obj}(S_1) \geq \text{Obj}(S_2) \quad (7)$$

A solution  $S_1$  is said to strongly dominate another solution  $S_2$  if for each objective it has a greater objective value.

$$\text{Obj}(S_1) > \text{Obj}(S_2) \quad (8)$$

The solution archive at a given generation of operation contains the set of non-dominated solutions found so far. A solution is said to be *Pareto optimal* if one of its objective values cannot be improved without simultaneously reducing another. If the solution archive grows larger than its maximum defined size then a diversity mechanism, as defined in [14], is utilized to retain the best spread of non-dominated solutions found.

### E. Multi-Objective Genetic Algorithms (GAs)

GAs, first developed by Holland [15], utilize nature's philosophies of genetics and natural selection to stochastically solve optimization problems - generally by means of evolving a population of potential solutions over many generations. GA can be used to solve both single and multi-objective problems. When we have multiple conflicting objectives there is usually not a single solution that optimizes all the objectives. Instead there are a set of solutions which, without additional information, are equally as suitable. The members of this set of solutions are termed *Pareto optimal solutions* and together they map out the trade-off boundary edge - the Pareto front - of the problem.

Various multi-objective GAs have been proposed, for example [14], [16]. They seek to find a set of solutions that are as close as possible to the problem's Pareto front whilst also being as evenly spread along the front as possible, creating an approximation of the problem's Pareto front.

Our approach utilizes the MOCeCell multiple objective GA [17]. In this methodology the population is arranged as a two-dimensional grid and an external archive is used to store the current best solutions found. Restrictive mating is utilized such that population individuals are selected to mate only with those individuals close to them in the grid. Binary Tournament is then utilized to perform this restricted selection process. This can ensure diversity is preserved within the population for longer. Additionally the mechanism of feedback is used to add a given number of the best solutions found so far back into the population at the start of each generation – so as to increase the aggressiveness towards finding Pareto optimal solutions. Utilizing a GA that makes use of an external archive and allowing this value to be user-specified, gives the DM control over the maximum number of solutions that may be returned and continues the philosophy of our approach to provide flexibility to the DM.

The benefits of our proposed MOO approach are three-fold. Firstly, it allows for a DM to state their preference to the type of inconsistency reduction of most concern to them. The DM can define whether to operate with respect to ordinal inconsistency, cardinal inconsistency or both. The DM also has control over the selection of measure(s) to be employed, regarding how changes in their judgments are calculated. Secondly, by adopting a MOO approach a range of possible trade-off solutions can be presented to the DM from which they can select their preferred solution. This also helps the DM glean knowledge regarding their problem and its inconsistency. Finally, the set of solutions presented to the DM preserve the original judgment scale used to define the initial judgments making it easier for the DM to assimilate and compare solutions.

#### IV. EXPERIMENTATION

The encoding of a GA's population defines how elements in the decision space are to be represented as individuals in the solution space. For our approach only the top triangle of a PCM is encoded as a numerical vector. By utilizing the reciprocal axiom [18], the rest of the PCM can be reconstructed from its top triangle. The number of judgments  $J$  needed to represent the top triangle of an  $n$  order PCM is given by:

$$J = \frac{n(n-1)}{2} \quad (9)$$

We can map the possible values of the bounded scale as a set of integers. Encoding in this way ensures that any solutions found will also conform to the original numerical judgment scale. Figure 1 shows an example encoding of an order 4 PCM  $O$  into an Encoded Judgment vector (EJ).

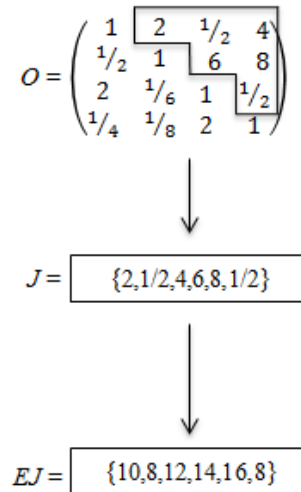


Figure 1. Example Encoding of a PCM

Our first example is a PCM taken from [19] (Table I) and the second is an example PCM with high levels of both cardinal and ordinal inconsistency (Table III). The MOCeCell algorithm approach was applied with the following parameter settings: population size of 100 (10x10 grid); maximum evaluations count of 25,000; archive size of 10 with a feedback value of 5. Selection is performed via binary tournament with single point crossover (with crossover probability 0.9) and bit flip mutation (with probability 0.01) employed. The size of the archive is re-definable by the DM.

#### Example 1: [19]

	1	2	3	4	5	6	7	8
1	1	5	3	7	6	6	1/3	1/4
2	1/5	1	1/3	5	3	3	1/5	1/7
3	1/3	3	1	6	3	4	6	1/5
4	1/7	1/5	1/6	1	1/3	1/4	1/7	1/8
5	1/6	1/3	1/3	3	1	1/2	1/5	1/6
6	1/6	1/3	1/4	4	2	1	1/5	1/6
7	3	5	1/6	7	5	5	1	1/2
8	4	7	5	8	6	6	2	1

Assuming the DM chooses objectives CR and STJD – NB: other objectives could be chosen instead of or along with these. Figure 2 shows a final solution space and solutions for CR against STJD. The CR threshold of 0.1 is shown via a dashed vertical line. The DM is then free to review and select any of these solutions. For instance, the DM could select the 1st solution along the Pareto front with a CR value less than 0.1 (Table II). The DM could equally have selected any other solution or remained with their original judgment set.

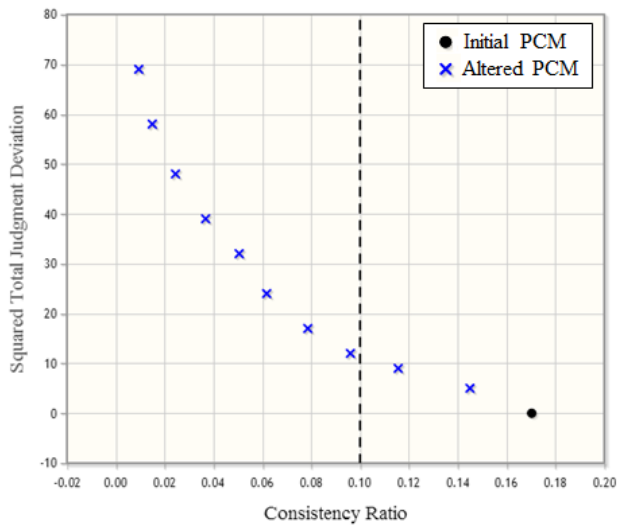


Figure 2. Example 1 Solution Space

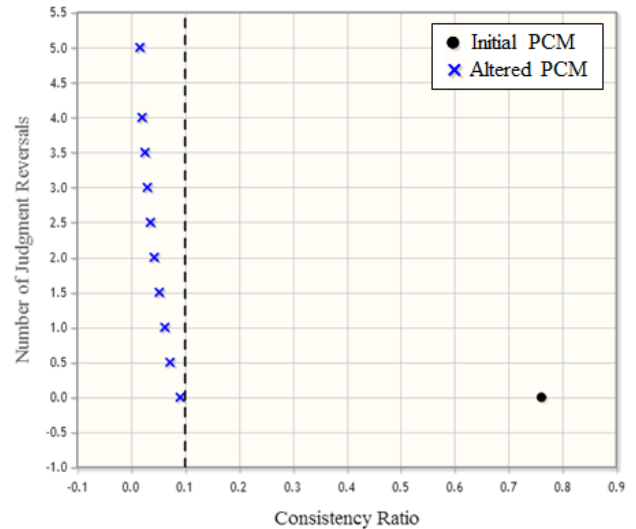


Figure 3. Example 2 Solution Space 1

TABLE II. POSSIBLE EXAMPLE 1 SOLUTION CR: 0.09, L: 0

	1	2	3	4	5	6	7	8
1	1	5	2	7	6	6	1	1/4
2	1/5	1	1/3	5	3	2	1/5	1/7
3	1/2	3	1	6	3	4	4	1/5
4	1/7	1/5	1/6	1	1/3	1/3	1/7	1/8
5	1/6	1/3	1/3	3	1	1/2	1/5	1/6
6	1/6	1/2	1/4	3	2	1	1/5	1/6
7	1	5	1/4	7	5	5	1	1/2
8	4	7	5	8	6	6	2	1

### Example 2: PCM with high levels of inconsistency

TABLE III. EXAMPLE 2: CR: 0.76 AND L: 9

	1	2	3	4	5	6	7	8	9
1	1	1/8	1/3	1/7	1/3	1	8	1/9	1/4
2	8	1	5	1/2	1/3	4	3	7	5
3	3	1/5	1	2	1/2	1/6	7	7	1/9
4	7	2	1/2	1	1	5	2	2	1/9
5	3	3	2	1	1	7	6	5	6
6	1	1/4	6	1/5	1/7	1	2	1/6	1
7	1/8	1/3	1/7	1/2	1/6	1/2	1	1	8
8	9	1/7	1/7	1/2	1/5	6	1	1	1/8
9	4	1/5	9	9	1/6	1	1/8	8	1

Assuming the DM is interested in seeking a reduction in cardinal inconsistency and is also concerned with minimizing NJR, they can select CR and NJR as their objectives. Figure 3 shows a range of solutions within the solution space found for these objectives.

The DM is able to select any solution ranging from 0 NJR upwards. They could choose a solution that has a large decrease in CR (to less than 0.1) and a NJR value of 0 – so without a single reversal or half reversal occurring (Table IV).

TABLE IV. POSSIBLE EXAMPLE 2 SOLUTION 1: CR: 0.09, L: 9

	1	2	3	4	5	6	7	8	9
1	1	1/3	1/2	1/2	1/6	1	2	1/2	1/3
2	3	1	2	1/2	1/2	2	4	2	2
3	2	1/2	1	2	1/4	1/2	2	2	1/2
4	2	2	1/2	1	1	2	3	2	1/2
5	6	2	4	1	1	5	6	5	3
6	1	1/2	2	1/2	1/5	1	2	1/2	1
7	1/2	1/4	1/2	1/3	1/6	1/2	1	1	2
8	2	1/2	1/2	1/2	1/5	2	1	1	1/2
9	3	1/2	2	2	1/3	1	1/2	2	1

Alternatively the DM may be more concerned in reducing ordinal inconsistency and thus instead selects objectives L and NJR. The associated solution space (Figure 4) shows that a solution has been found with an NJR value of 3.5 with all 3-way cycles removed (Table V). The DM is free to choose this solution or any other solution of varying amounts of NJR.

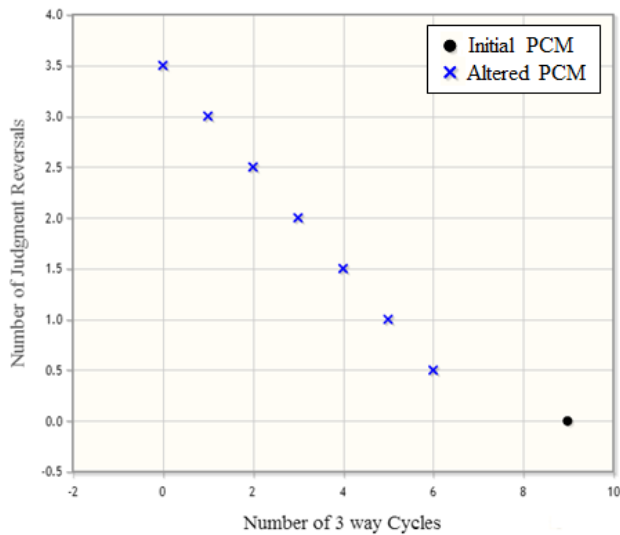


Figure 4. Example 2 Solution Space 2

TABLE V. POSSIBLE EXAMPLE 2 SOLUTION 2, CR: 0.33 L: 0

	1	2	3	4	5	6	7	8	9
1	1	1/7	1/4	1/9	1/7	1	7	1/5	1/7
2	7	1	6	1/7	1/9	4	2	3	1
3	4	1/6	1	1/2	1/4	1/3	5	1	1/6
4	9	7	2	1	1/4	6	9	8	1/7
5	7	9	4	4	1	3	6	4	9
6	1	1/4	3	1/6	1/3	1	6	1/6	1
7	1/7	1/2	1/5	1/9	1/6	1/6	1	1	1/5
8	5	1/3	1	1/8	1/4	6	1	1	1/3
9	7	1	6	7	1/9	1	5	3	1

## V. CONCLUSION AND FUTURE WORK

We have presented a MOO approach to reducing inconsistency within PCMs. The approach gives the DM control over the objectives chosen regarding both the type of inconsistency reduction and the measures of change to their judgments. By employing a multi-objective approach, a range of possible solutions can be presented to the DM for comparison and selection. The DM can also control the total number of possible solutions that may be returned. The presented solutions maintain the original judgment scale utilized.

Future work will provide further flexibility by, for example, allowing the DM to define soft or hard constraints upon the objectives - producing more aggressive searching within certain areas of the solution space. GA performance will be analyzed with regards to quality, speed and stability of produced results. Visualization of the solution space with more objectives will also be investigated.

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